

# New results for the Inert Doublet Model\*

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## Abstract

New results for the Inert Doublet Model (IDM) are discussed. It is very special among the  $D$ -symmetric 2HDMs, offering a good DM candidate. New unitarity constraints were derived for the IDM and SM-like light Higgs boson scenario in the Mixed Model.

## 1 Two Higgs Doublet Models

Among the standard models of the elementary particle interactions the most popular are the one with one Higgs (scalar) doublet (The Standard Model SM = 1HDM) and with two such doublets (2HDMs, including Minimal Supersymmetric Standard Model (MSSM)). In 2HDMs there are five scalars - three neutral and two charged ones. The lightest neutral scalar is often SM-like, what makes such models particularly interesting nowadays.

The Brout-Englert-Higgs mechanism describing spontaneous breaking of the EW symmetry allows in 2HDM's for breaking of the  $U(1)_{QED}$  symmetry, in contrast to the 1HDM. In these models two scalar doublets of SU(2), with weak hypercharge equal 1, can be involved in generating masses of the gauge bosons  $W^\pm$  and  $Z$ . Fermion masses are generated via Yukawa interactions in various ways, leading to various models: Model I, II, III, IV, X, Y, ... depending on how the doublets couple to fermions. Typically, in order to avoid FCNC at the tree level, some discrete symmetries are imposed on a Lagrangian. Here we will consider the Lagrangian, which is symmetric under such  $Z_2$  transformation, where one of the scalar doublets changes sign, while all other fields (the other scalar doublet and all SM-fields) are unchanged. This allows us to consider a case of the Inert Doublet Model (IDM), in which such a  $Z_2$  symmetry is respected not only at the Lagrangian level but also in the vacuum [1]. IDM is unique among 2HDMs as

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it predicts existence of a stable particle - a good candidate for the dark matter (DM) [2, 3, 4, 5, 6].

We will call the scalar doublet which changes sign under the transformation  $\phi_D$ , while the other scalar doublet we will denote as  $\phi_S$ . The corresponding  $Z_2$ -symmetry will be called the  $D$ -symmetry. The scalars will be universally denoted by  $h, H, A, H^\pm$  in all 2HDMs considered here.

We can consider the following  $D$ -symmetric potential [7]:

$$\begin{aligned} V = & -\frac{1}{2} \left[ m_{11}^2 (\phi_S^\dagger \phi_S) + m_{22}^2 (\phi_D^\dagger \phi_D) \right] + \\ & + \frac{\lambda_1}{2} (\phi_S^\dagger \phi_S)^2 + \frac{\lambda_2}{2} (\phi_D^\dagger \phi_D)^2 + \lambda_3 (\phi_S^\dagger \phi_S) (\phi_D^\dagger \phi_D) + \\ & + \lambda_4 (\phi_S^\dagger \phi_D) (\phi_D^\dagger \phi_S) + \frac{\lambda_5}{2} \left[ (\phi_S^\dagger \phi_D)^2 + (\phi_D^\dagger \phi_S)^2 \right], \end{aligned} \quad (1a)$$

with all parameters real and with an additional condition  $\lambda_5 < 0$ . The IDM is realized in some regions of parameter space of this potential. We will consider also other possible vacuum states of such potential, realized at another values of parameters. This allows to consider possible temperature evolutions of vacua and transitions between them, see below and in [7, 6].

## 2 Extrema and vacua

Extrema of the 2HDM potential with an explicit  $D$ -symmetry can be found as usual: first, one finds extrema, then minima and then the global minimum, which is the vacuum. Positivity (stability) constraints on the  $V$  are as follows

$$\lambda_1 > 0, \lambda_2 > 0, R + 1 > 0, \lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, R = \lambda_{345} / \sqrt{\lambda_1 \lambda_2}. \quad (2)$$

The *extremum* respecting these constraints, which has the lowest energy, is the vacuum of the system [7].

In general the extrema have the following form:

$$\langle \Phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \quad \langle \Phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_D \end{pmatrix}, \quad (3)$$

with  $v_S > 0$  and  $v^2 = v_S^2 + |v_D|^2 + u^2$ ,  $v=246$  GeV. Properties of extrema respecting and violating  $U(1)_{QED}$  symmetry ( $u = 0$  and  $u \neq 0$ , respectively) are presented in Table 1.

It is very useful to represent extrema in the  $(\lambda_4, \lambda_5)$  plane (Fig. 1) [8]. In this figure positivity constraints lead to the bounds:  $\lambda_4 \pm \lambda_5 > -X$ , where  $X = \sqrt{\lambda_1 \lambda_2} + \lambda_3 > 0$ ; for the Inert vacuum  $Y = 2M_{H^\pm}^2 |_{Inert}^2 / v^2 > 0$ , while for the Charged one  $\lambda_4 \pm \lambda_5 > 0$ .<sup>1</sup> Note the overlap of the regions where the Inert (or Inert-like) vacuum can be realized with the corresponding ones allowing for Mixed and/or Charged vacua.

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<sup>1</sup>Here we show  $\lambda_5 > 0$  regions allowed for Inert(Inert-like) and Charged vacua, which are symmetric to the  $\lambda_5 < 0$  ones, with a change of roles of  $H$  and  $A$  particles.

Extrema	
<i>EW symmetric: EWs</i>	$u = 0, \quad v_D = 0, \quad v_S = 0$
<i>Inert: <math>I_1</math></i>	$u = 0, \quad v_D = 0, \quad v_S^2 = \frac{m_{11}^2}{\lambda_1}$
<i>Inert-like: <math>I_2</math></i>	$u = 0, \quad v_S = 0, \quad v_D^2 = \frac{m_{22}^2}{\lambda_2}$
<i>Mixed: <math>M</math></i>	$u = 0, \quad v_S^2, v_D^2 > 0$
<i>Charged: <math>Ch</math></i>	$u, v_S^2 > 0$

Table 1: General properties of extrema, following [7].

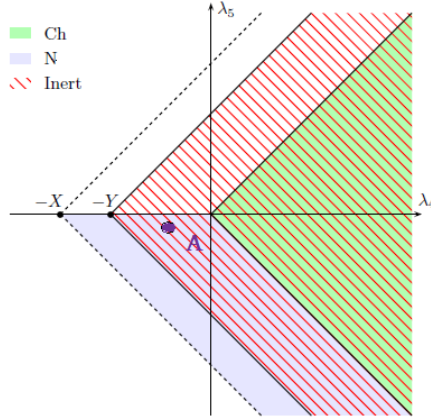


Figure 1: Region allowed by the positivity constraints (on the right of the dotted lines). Allowed regions for extrema: the Inert (Inert-like) (hatched area), Mixed (shaded area for  $\lambda_4 + \lambda_5 < 0$ ) and Charged (hatched shaded area for  $\lambda_4 \pm \lambda_5 > 0$ ). Point A corresponds to a possible today's Universe state.

If Nature is described today by the  $D$ -symmetric 2HDM Lagrangian (with Model I of the Yukawa interactions, where only the  $\phi_S$  couples to fermions) then the question is, which vacuum (phase) is realized today? Definitely charge breaking phase is not a good candidate, as here photon would be massive and electric charge would not be conserved. Among neutral phases only the Inert one, being in agreement with accelerator and astrophysical data, offers a good neutral DM candidate (for  $\lambda_5 < 0$  it is a  $H$ ). Inert-like phase is excluded as here all the fermions would be massless, on the other hand Mixed phase is in agreement with the accelerator data.

We have considered evolution of the Universe in 2HDM, during its cooling down from the EW symmetric phase to the present Inert phase [7], see also [9]. For this purpose thermal evolution of the explicitly  $D$ -symmetric Lagrangian was considered in the simplest approximation, where only mass terms in  $V$  vary with temperature like  $T^2$ , while parameters  $\lambda$ 's are fixed. In Fig.1 we show a possible position of the today's Universe. In the past it could go through various phases in one, two or three phase transitions. We found that the first phase transitions (i.e. from the EWs phase) are all of the 2nd order. One should, however, consider other thermal corrections beyond the  $T^2$  approximation; preliminary results were obtained recently [10], suggesting that the type of these transitions may change.

### 3 Inert Doublet Model

In the IDM  $D$ -symmetry is conserved and one can assign  $D$ -parity to all particles. The  $\phi_S$ , with  $D$ -parity even, plays a role of Higgs doublet in the SM, with one Higgs (SM-like) particle  $h$  (with  $M_h^2 = \lambda_1 v^2 = m_{11}^2$ ). The second doublet  $\phi_D$ , with zero vacuum expectation value, is  $D$ -odd, it contains 4 scalars (not Higgs particles!) and the lightest particle among them is stable and can be a dark matter particle. We call all these scalars - dark scalars; their masses are given by

$$M_{H^\pm}^2 = \frac{\lambda_3 v^2 - m_{22}^2}{2}, M_A^2 = M_{H^\pm}^2 + \frac{\lambda_4 - \lambda_5}{2} v^2, M_H^2 = M_{H^\pm}^2 + \frac{\lambda_4 + \lambda_5}{2} v^2. \quad (4)$$

Couplings among scalars are given by  $\lambda$ 's:  $\lambda_1$  is proportional to  $hhh$  coupling and fixed by the mass of  $h$ ,  $\lambda_{345}$  describes trilinear couplings of  $h$  with dark scalars, while  $\lambda_2$  appears only in quartic selfcouplings of the dark scalars.

Theoretical constraints of the IDM arising from the positivity (stability) condition and conditions for the Inert vacuum were discussed above and can be found in [7, 6]. The important new unitarity constraints were obtained in [12] and are presented below. Here we would like to mention agreement of this model with precision EW data, in form of  $S, T, U$ , see eg. [2, 3, 5].

Phenomenologically, testing the IDM at present and future colliders can be performed by precise measurements of properties of the SM-like  $h$  and by direct search of dark scalars' pairs. There exist some constraints from LEP II (masses  $H$  versus  $A$ ) [11], as well as analysis on DM [4, 5, 6].

## 4 Unitarity constraints for the IDM and the Mixed Model

Here we present new results on unitarity constraints for the  $D$ -symmetric 2HDM potential [12]. It updates the previous analyses for parameters  $\lambda$  [13]. We have applied the standard approach [14], using equivalence theorem to deal with Goldstone bosons instead of longitudinal gauge bosons and neglecting the trilinear couplings. We applied, for the first time, unitarity constraints for the IDM and for the SM-like scenario within the Mixed Model (based on the Mixed vacuum and the Model II of Yukawa interactions).

Full tree-level high energy scattering matrix (of dimension 25) for the scalars was considered, including the double charged initial/final states not studied previously. Diagonalization thereof leads to 12 distinct eigenvalues being functions of the quartic couplings (or equivalently of the parameters  $\lambda$ ) [15, 16]. Applying the standard unitarity condition  $|\Re(a^{(j)}(s))| \leq \frac{1}{2}$  to these eigenvalues yields a set of inequalities for  $\lambda$ 's or, if different set of parameters is chosen, for the masses of scalar particles. These inequalities were solved numerically, probing statistically a large range of values of the parameters (as in [16]) and taking into account the vacuum stability conditions and conditions determining the type of vacuum, as discussed in Sec. 3. The results of the scans give bounds on the values of the  $\lambda$  parameters and masses and correlations between them (see Fig. 2), not considered in previous analyses [15, 16]. The constraints obtained for the  $\lambda$ 's are more stringent than the previous ones and read:

$$\begin{aligned} 0 &\leq \lambda_1 \leq 8.38, \\ 0 &\leq \lambda_2 \leq 8.38, \\ -5.85 &\leq \lambda_3 \leq 16.33, \\ -15.82 &\leq \lambda_4 \leq 5.93, \\ -8.21 &\leq \lambda_5 \leq 0. \end{aligned} \tag{5}$$

Similarly, the following combinations of  $\lambda$ 's are constrained ( $\lambda_{ij} = \lambda_i + \lambda_j$ )

$$\begin{aligned} -7.90 &\leq \lambda_{345} \leq 11.31, \\ -16.37 &\leq \lambda_{45} \leq 0, \\ -7.45 &\leq \lambda_{34} \leq 12.55, \end{aligned} \tag{6}$$

which in the IDM correspond directly to the bounds on quartic couplings between physical fields <sup>2</sup>.

In the IDM the bounds on masses of the scalars depend on one additional parameter  $m_{22}^2$  (Eq.(3)). However, this dependence is negligible for  $|m_{22}^2| \leq 10^4 \text{ GeV}^2$ . The upper bounds on masses of  $H^\pm$  and  $A$  for this case (when  $M_h = 120 \text{ GeV}$  or  $M_h \in [114, 145] \text{ GeV}$ ) are of order of 700 GeV and for  $M_H$  of order of 600 GeV. This shows that in a wide region of values of  $m_{22}^2$  the possibility of existence of a very heavy (with mass over 800 GeV) dark matter

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<sup>2</sup>  $\lambda_{345}$  represents a coupling of  $hhHH$ ,  $AAGG$ ,  $\lambda_{45}$  is a coupling of a vertex containing  $H^+G^-$  or  $G^+H^-$  and  $\lambda_{34}$  is a coupling of  $G^+G^-H^+H^-$

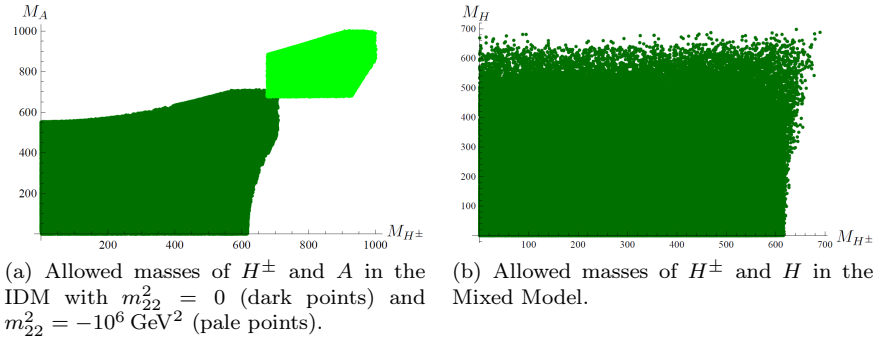


Figure 2: Regions of masses allowed by the unitarity constraint for the IDM (left) and the Mixed Model (right).

particle is excluded. The region of masses allowed by the unitarity condition for the cases with  $m_{22}^2 = 0$  and  $m_{22}^2 = -10^6 \text{ GeV}^2$  is shown in Fig. 2a.

In the Mixed Model the upper bounds for the heavy scalars' ( $H^\pm$ ,  $H$ ,  $A$ ) masses are of order of 700 GeV and for the  $h$  boson of order of 500 GeV (as in [17]). The region of masses of  $H^\pm$  and  $H$  allowed by the unitarity condition is shown in Fig. 2b.

In addition, we consider the SM-like scenario of the Mixed Model, with the condition  $\sin(\beta - \alpha) = 1$ . Then the  $h$  boson couples (at the tree-level) to fermions and gauge bosons exactly as the SM Higgs boson and the experimental constraints for the SM Higgs mass can be applied to  $h$ :  $M_h \in [114, 145] \text{ GeV}$ . Unitarity constraints lead to the upper bounds of about 600 GeV for  $M_H$  and  $M_{H^\pm}$ , which are lowered as compared to the arbitrary  $\sin(\beta - \alpha)$  case, and do not bound  $M_h$  any further.

## 5 Conclusions

A significant progress has been obtained recently in understanding the underlying structure of the simple extensions of the SM with two scalar doublets. IDM is very special among the  $D$ -symmetric 2HDMs, offering a good DM candidate. New unitarity constraints were derived for the IDM and SM-like light Higgs boson scenario in the Mixed Model.

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## References

- [1] N. G. Deshpande and E. Ma, Phys. Rev. D **18** (1978) 2574.
- [2] R. Barbieri, L. J. Hall, V. S. Rychkov, Phys. Rev. D **74** (2006) 015007.
- [3] Q. H. Cao et al. Phys. Rev. D **76** (2007) 095011 [arXiv:0708.2939 [hep-ph]].
- [4] L. Lopez Honorez, et al. JCAP **0702** (2007) 028 [arXiv:hep-ph/0612275].
- [5] E. M. Dolle, S. Su, Phys. Rev. D **80** (2009) 055012.
- [6] D. Sokołowska, *Dark Matter data and quartic self-couplings in Inert Doublet Model*, this proceeding; arXiv:1107.1991v1 [hep-ph].
- [7] I. F. Ginzburg, *et al.*, Phys. Rev. D **82** (2010) 123533.
- [8] M. Krawczyk, D. Sokołowska, arXiv:0911.2457 [hep-ph].
- [9] I. P. Ivanov, Acta Phys. Polon. B **40** (2009) 2789; I. F. Ginzburg, *et al.*, Phys. Rev. D **81** (2010) 085031.
- [10] G. Gil, *Study of the phase transitions in the Inert Doublet Model*, Master Thesis (in Polish), University of Warsaw, August 2011.
- [11] E. Lundstrom, M. Gustafsson, J. Edsjo, Phys. Rev. D **79** (2009) 035013.
- [12] B. Gorczyca, *Unitarity constraints for the Inert Doublet Model*, Master Thesis (in Polish), University of Warsaw, July 2011.
- [13] H. Hüffel, G. Pócsik, Z. Phys. C - Particles and Fields **8** (1981) 13-15.
- [14] B. W. Lee, C. Quigg, H. B. Thacker, Phys. Rev. D **16** (1977) 1519.
- [15] S. Kanemura, T. Kubota, E. Takasugi, Phys. Lett. B **313** (1993) 155-160.
- [16] A. G. Akeroyd, A. Arhrib, E. Naimi, Phys. Lett. B **490** (2000) 119-124.
- [17] J. Hořejší, M. Kladiva, Eur. Phys. J. C **46** (2006) 81.
- [18] M. Misiak *et al.*, Phys. Rev. Lett. **98** (2007) 022002.